02

In previous chapter, we have studied about the existence of inverse of a function. In this chapter, we shall study about the restrictions on domains and ranges of trigonometric functions, which ensure the existence of their inverse and observe their behaviour through graphical representations.

INVERSE TRIGONOMETRIC FUNCTIONS

CHAPTER CHECKLIST

- Inverse of a Function
- Inverse Trigonometric Functions

INVERSE OF A FUNCTION

We know that if a function is one-one and onto, then it is an invertible function. Suppose $f: X \to Y$ such that $f(x) = y$ is one-one and onto, then we define a unique function $g: Y \to X$ such that $g(y) = x$, where $x \in X$ and $y \in Y$.

Here, g is called the inverse of f and it is denoted by 'f^{-1}'.

Further, g is also one-one and onto, thus $g = f^{-1}: Y \to X$.

Clearly, domain of f^{-1} = range of f and range of f^{-1} = domain of f. Before we study the inverse trigonometric function, we have to know the domain and range of trigonometric function, which are shown below

Function	Domain	Range
$\sin x$	R	$[-1, 1]$
$\cos x$	R	$[-1, 1]$
$\tan x$	$R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$	R
$\cot x$	$R - \{x : x = n\pi, n \in Z\}$	R
$\sec x$	$R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$	$R - (-1, 1)$
$\operatorname{cosec} x$	$R - \{x : x = n\pi, n \in Z\}$	$R - (-1, 1)$

INVERSE TRIGONOMETRIC FUNCTIONS

As we know that trigonometric functions are periodic functions, so these functions are many-one. Trigonometric functions are not one-one and onto over their natural domain and range, so their inverse do not exist, but if we

Function $\sin^{-1} x$, whose domain is $[-1, 1]$ and range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, is written as $\sin^{-1} x : [-1, 1] \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

Graph of Inverse Trigonometric Functions

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restrict their domain and range, then their inverse may exists.

Domain and Range of Inverse Trigonometric Functions

The range of trigonometric functions becomes the domain of inverse trigonometric functions and restricted domain of trigonometric functions becomes **range** or **principal value branch** of inverse trigonometric functions.

e.g. Let the function $f : R \to R$ defined as $f(x) = \sin x$.

Since, the domain of sine function is a set of all real numbers and range $[-1, 1]$. Therefore, it is many-one function. If we restrict its domains to anyone of the intervals $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, etc., then it becomes one-one onto and in each case and the range is $[-1, 1]$.

Therefore, we can define the inverse of sine function in each of these intervals and denoted by $\sin^{-1}$ (arc sine function). Thus, $\sin^{-1}$ is a function whose domain is $[-1, 1]$ and range may be any of the intervals $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ and so on. Corresponding to each such interval, we get branch of function $\sin^{-1}$.

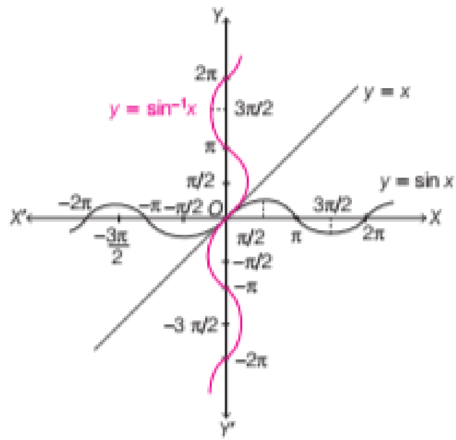
The branch with range $[-\pi/2, \pi/2]$ is called principal value branch and value belonging to it is called principal value.

The graph of an inverse trigonometric functions can be obtained from the graph of trigonometric functions in two ways

(i) By interchanging X and Y-axes, i.e. if (a, b) is a point on the graph of trigonometric function, then (b, a) becomes the corresponding point on the graph of inverse trigonometric functions.

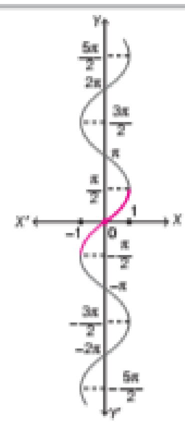
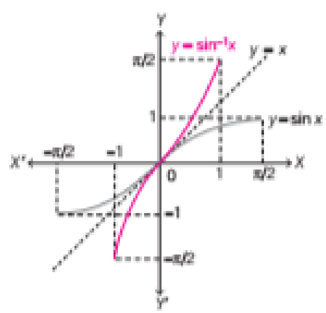
(ii) As a mirror image (i.e. reflection) along the line $y = x$.

e.g. The graph of inverse sine function is obtained from the corresponding graph of original sine function.



It is clear from the graph that inverse of sine function is the mirror image of sine function with respect to the line $y = x$.

Domain, Principal Value Branch (Range) **and Graphs of Standard Inverse Trigonometric Function**

Function	Domain	Principal value branch (Range)	Other possible range	Graph (By interchanging axes)	Graph (As mirror image)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, etc.		

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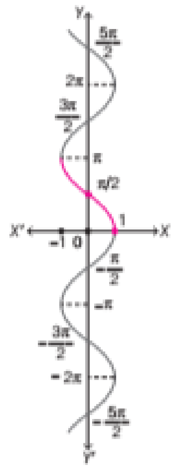
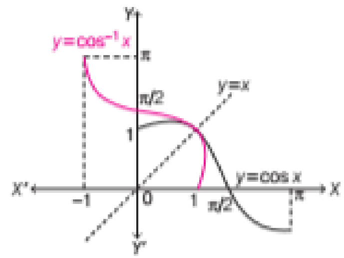
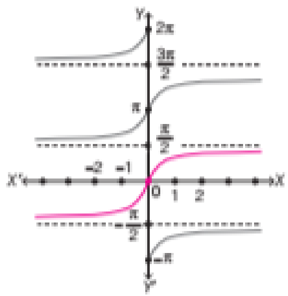
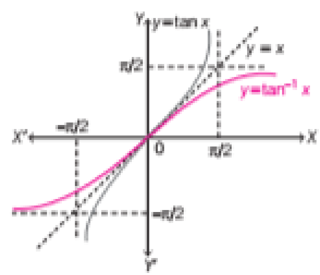
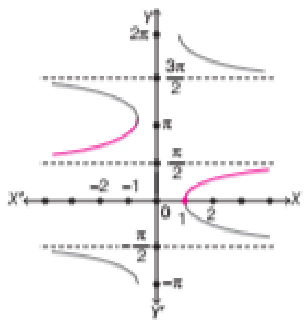
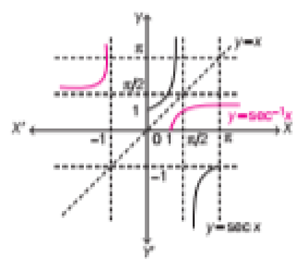
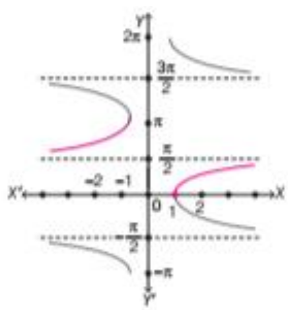
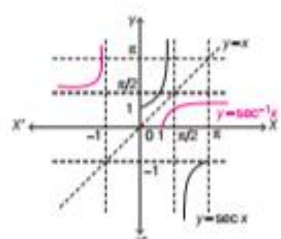
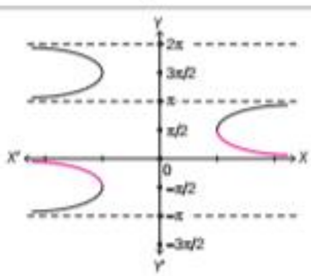
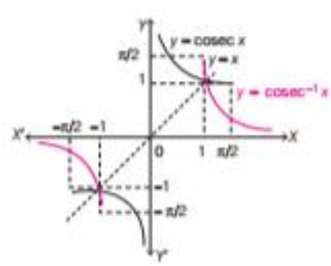
Function	Domain	Principal value branch (Range)	Other possible range	Graph (By interchanging axes)	Graph (As mirror image)
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	$[-\pi, 0]$, $[\pi, 2\pi]$, etc.		
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$, $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ etc.		
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	$[-\pi, 0] - \left\{-\frac{\pi}{2}\right\}$, $[\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$, etc.		
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	$[-\pi, 0] - \left\{-\frac{\pi}{2}\right\}$, $[\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$, etc.		
$y = \text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	$\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right] - \{-\pi\}$, $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$, etc.		

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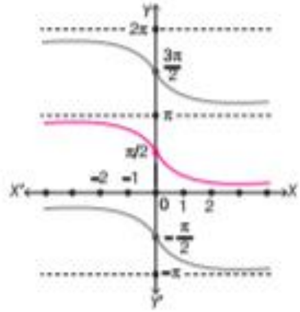
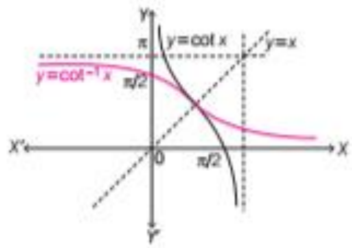
Function	Domain	Principal value branch (Range)	Other possible range	Graph (By interchanging axes)	Graph (As mirror image)
$y = \cot^{-1} x$	R	$(0, \pi)$	$(-\pi, 0), (\pi, 2\pi)$, etc.		

Note In the above graphs, coloured line denotes the graph of inverse trigonometric functions in the principal value branch.

IMPORTANT POINTS RELATED TO INVERSE TRIGONOMETRIC FUNCTIONS

(i) $\sin^{-1} y \neq (\sin y)^{-1}$, $\sin^{-1} y \neq \sin^{-1}\left(\frac{1}{y}\right)$,

$\sin^{-1} y \neq \frac{1}{\sin y}$. These relations are also hold for other inverse trigonometric functions.

(ii) Whenever no branch of an inverse trigonometric function is mentioned, we consider the principal value branch of that function.

(iii) If $\sin^{-1} x = y$, then x and y are the elements of domain and range of principal value branch of $\sin^{-1} x$, respectively.

(iv) In the graph of inverse function, if we draw a vertical line, then it cut the graph at many points which shows that it is not a function. That's why we restricts the domain for inverse trigonometric functions.

Problems Based on Principal Value Branch of Inverse Trigonometric Functions

There are mainly two types of problems. In first type of problem, we have to find the principal value of given inverse trigonometric function and in second type of problem, we have to find the value of two or more than two inverse trigonometric functions, which are connecting with operations '+' or '–'.

For solving such problems, we take value of each inverse trigonometric function in its principal value branch.

EXAMPLE |1| Find the domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$. **[NCERT Exemplar]**

Sol. Given function, $f(x) = \sin^{-1}\sqrt{x-1}$

For domain of $f(x)$, $0 \leq \sqrt{x-1} \leq 1$

$\Rightarrow \quad 0 \leq x - 1 \leq 1 \Rightarrow 1 \leq x \leq 2$

$\therefore \quad x \in [1, 2]$

EXAMPLE |2| Find the principal value of the following.

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}\left(-\frac{1}{2}\right)$ (iii) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ **[NCERT]**

Sol. (i) Let $y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \sin y = \frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4}\right) \Rightarrow y = \frac{\pi}{4}$ $\left[\because \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$

We know that the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$.

(ii) Let $\sin^{-1}\left(-\frac{1}{2}\right) = \theta \Rightarrow \sin\theta = -\frac{1}{2}$

$\therefore \sin\theta = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

$[\because \sin(-\theta) = -\sin\theta]$

$\Rightarrow \quad \theta = -\frac{\pi}{6}$

We know that the range of principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$.

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(iii) Let $y = \cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) \Rightarrow \cot y = \dfrac{-1}{\sqrt{3}}$

$\Rightarrow \cot y = -\cot\dfrac{\pi}{3} = \cot\left(\pi - \dfrac{\pi}{3}\right)$

$[\because \cot(\pi - \theta) = -\cot\theta]$

$\Rightarrow \quad \cot y = \cot\left(\dfrac{2\pi}{3}\right) \Rightarrow y = \dfrac{2\pi}{3} \in (0, \pi)$

[$\because$ principal branch of $\cot^{-1}$ is $(0, \pi)$]

EXAMPLE |3| Find the value of $\tan^{-1}(-1)$ in the interval $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$.

Sol. Let $y = \tan^{-1}(-1) \Rightarrow \tan y = -1$

$\Rightarrow \quad \tan y = \tan\dfrac{3\pi}{4} \quad \left[\because \dfrac{3\pi}{4} \in \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)\right]$

$\Rightarrow \quad y = \dfrac{3\pi}{4}$

EXAMPLE |4| Find the value of $\sin\left[\dfrac{\pi}{3} - \sin^{-1}\left(-\dfrac{1}{2}\right)\right]$.

Sol. $\sin\left[\dfrac{\pi}{3} - \sin^{-1}\left(-\dfrac{1}{2}\right)\right]$

$= \sin\left[\dfrac{\pi}{3} + \dfrac{\pi}{6}\right] \quad \left[\because \sin^{-1}\left(\dfrac{-1}{2}\right) = \dfrac{-\pi}{6}\right]$

$= \sin\dfrac{\pi}{2} = 1$

EXAMPLE |5| Evaluate $\cos^{-1}\left(\dfrac{1}{2}\right) + 2\sin^{-1}\left(\dfrac{1}{2}\right)$.

[NCERT; All India 2013]

Sol. Let $\cos^{-1}\left(\dfrac{1}{2}\right) = x$

$\Rightarrow \quad \cos x = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

$\Rightarrow \quad x = \dfrac{\pi}{3} \in [0, \pi]$

[$\because$ principal value branch of $\cos^{-1}$ is $[0, \pi]$]

Again, let $\sin^{-1}\left(\dfrac{1}{2}\right) = y$

$\Rightarrow \quad \sin y = \dfrac{1}{2} = \sin\dfrac{\pi}{6} \Rightarrow y = \dfrac{\pi}{6} \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\left[\because \text{principal value branch of } \sin^{-1} \text{ is } \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\therefore \cos^{-1}\left(\dfrac{1}{2}\right) + 2\sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3} + 2\cdot\dfrac{\pi}{6}$

$= \dfrac{\pi}{3} + \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

EXAMPLE |6| Find the value of $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) + \tan^{-1}(\sqrt{3})$.

Sol. For principal value of $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$.

Let $y = \sin^{-1}\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin y = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin y = \sin\dfrac{\pi}{3} \quad \left[\because \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}\right]$

$\Rightarrow \quad y = \dfrac{\pi}{3} \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\left[\because \text{principal value branch of } \sin^{-1} \text{is} \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\therefore \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$

For principal value of $\tan^{-1}(\sqrt{3})$,

Again, let $x = \tan^{-1}(\sqrt{3})$

$\Rightarrow \quad \tan x = \sqrt{3}$

$\Rightarrow \quad \tan x = \tan\left(\dfrac{\pi}{3}\right) \quad \left[\because \tan\dfrac{\pi}{3} = \sqrt{3}\right]$

$\Rightarrow \quad x = \dfrac{\pi}{3} \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$\left[\because \text{principal value branch of } \tan^{-1} \text{is} \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)\right]$

$\Rightarrow \tan^{-1}(\sqrt{3}) = \dfrac{\pi}{3}$

$\therefore \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) + \tan^{-1}(\sqrt{3}) = \dfrac{\pi}{3} + \dfrac{\pi}{3}$

$= \dfrac{2\pi}{3}$

EXAMPLE |7| Find the value of $\tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\dfrac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\dfrac{\pi}{2}\right)\right]$.

Sol. $\tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\dfrac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\dfrac{\pi}{2}\right)\right]$

$= \dfrac{-\pi}{6} + \dfrac{\pi}{3} + \tan^{-1}(-1)$

$\left[\because \tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = \dfrac{-\pi}{6}, \cot^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = \dfrac{\pi}{3}, \sin\left(\dfrac{-\pi}{2}\right) = -1\right]$

$= \dfrac{-\pi}{6} + \dfrac{\pi}{3} - \dfrac{\pi}{4}$

$= \dfrac{-\pi}{12}$

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EXAMPLE |8| Find the value of

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right].$$

Sol. Given, $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$$= \tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(2\sin\frac{\pi}{3}\right)$$

$$= \tan^{-1}\left(2\times\frac{\sqrt{3}}{2}\right)$$

$$= \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

EXAMPLE |9| Find the value of

$$\cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}(-\sqrt{3}) - \text{cosec}^{-1}(2).$$

Sol. Firstly, assume the given inverse trigonometric function equal to y (or x).

Let $y = \cos^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow \cos y = \frac{-1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$

$[\because \cos(\pi - \theta) = -\cos\theta]$

$\Rightarrow \cos y = \cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow \quad y = \frac{2\pi}{3}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\frac{2\pi}{3} \in [0, \pi]$.

So, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Now, find the principal value of $\tan^{-1}(-\sqrt{3})$.

Let $x = \tan^{-1}(-\sqrt{3})$

$\Rightarrow \quad \tan x = -\sqrt{3} = -\tan\frac{\pi}{3} \qquad \left[\because \tan\frac{\pi}{3} = \sqrt{3}\right]$

$\Rightarrow \quad \tan x = \tan\left(\frac{-\pi}{3}\right) \qquad [\because \tan(-\theta) = -\tan\theta]$

$\Rightarrow \quad x = -\frac{\pi}{3}$

Since, the principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $-\frac{\pi}{3} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

So, the principal value of $\tan^{-1}(-\sqrt{3})$ is $\left(-\frac{\pi}{3}\right)$.

Now, find the principal value of $\text{cosec}^{-1}(2)$.

Let $z = \text{cosec}^{-1}(2)$

$\Rightarrow \text{cosec}\, z = 2 = \text{cosec}\frac{\pi}{6}$

$\Rightarrow \quad z = \frac{\pi}{6}$

Since, the principal value branch of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and $\frac{\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

So, the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}(-\sqrt{3}) - \text{cosec}^{-1}2$

$$= \frac{2\pi}{3} - \frac{\pi}{3} - \frac{\pi}{6}$$

$$= \frac{(4-2-1)\pi}{6}$$

$$= \frac{\pi}{6}$$

SUMMARY

- **Inverse of a Function** Suppose $f: X \longrightarrow Y$ such that $f(x) = y$ is one-one and onto, then we define a new function $g: Y \to X$ such that $g(y) = x$, where $x \in X$ and $y \in Y$. Here, g is called the inverse of f and it is denoted by f^{-1}.
- **Domain and Range of Inverse Trigonometric Functions** The range of trigonometric functions becomes the domain of inverse trigonometric functions and restricted domain of trigonometric functions becomes range or principal value branch of trigonometric functions.

Function	Domain	Principal Value Branch (Range)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \cot^{-1} x$	R	$(0, \pi)$

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- **Graph of Inverse Trigonometric Functions** The graph of inverse trigonometric functions can be obtained from the graph of trigonometric functions in following two ways
  (i) By interchanging X and Y-axes, i.e. if (a, b) is a point on the graph of trigonometric function, then (b, a) becomes the corresponding point on the graph of inverse trigonometric functions.
  (ii) As a mirror image (i.e. reflection) along the line $y = x$.

CHAPTER PRACTICE

OBJECTIVE TYPE QUESTIONS

1 The inverse of cosine function is defined in the intervals

(a) $[-\pi, 0]$ (b) $\left[\frac{-\pi}{2}, 0\right]$ (c) $\left[0, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}, \pi\right]$

2 The domain in which sine function will be one-one, is

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

(c) $[0, \pi]$ (d) Both 'a' and 'b'

3 If $\tan^{-1} x = y$, then **[CBSE 2021 (Term I)]**

(a) $-1 < y < 1$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

(c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $y \in \left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

4 If $\sin^{-1} x = y$, then

(a) $0 \le y \le x$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

(c) $0 < y < \pi$ (d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

5 The principal value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is **[CBSE 2021 (Term I)]**

(a) $\frac{\pi}{12}$ (b) π (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

6 The principal value of $[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})]$ is **[CBSE 2021 (Term I)]**

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

7 If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is

(a) 0 (b) 1 (c) 2 (d) 3

VERY SHORT ANSWER Type Questions

8 Evaluate $\cot^{-1}(-\sqrt{3})$. **[Delhi 2013]**

9 Write the value of $\tan^{-1}\left[\tan\left(\frac{15\pi}{4}\right)\right]$.

10 Find the principal value of the following. (Each part carries 1 Mark)

(i) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (ii) $\cot^{-1}\sqrt{3}$

(iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$ (iv) $\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(v) $\operatorname{cosec}^{-1}(2)$ (vi) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(vii) $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (viii) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **[NCERT]**

11 Find the domain of the function $\cos^{-1}(2x - 1)$. **[NCERT Exemplar]**

12 Find the principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$. **[NCERT Exemplar]**

13 Write the value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. **[Foreign 2014]**

14 Write the principal value of $\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)\right]$ **[Delhi 2013]**

SHORT ANSWER Type Questions

15 Find two branches other than the principal value branch of $\tan^{-1} x$.

16 Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$. **[NCERT]**

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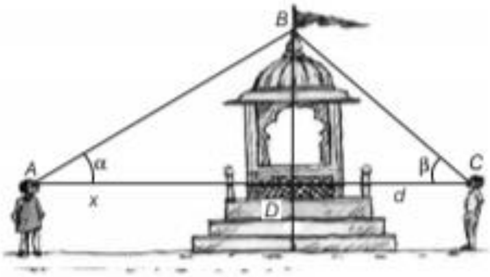
17 Find the value of $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$. [NCERT]

18 Find the value of $2\sec^{-1}2+\sin^{-1}\left(\frac{1}{2}\right)$. [NCERT Exemplar]

19 If $\tan^{-1}(\sqrt{3})+\cot^{-1}x=\frac{\pi}{2}$, then find x.

CASE BASED Questions

20 Two men on either side of a temple of 30 m high observe its top at the angles of elevation α and β, respectively (as shown in the figure below).



The distance between the two men is $40\sqrt{3}$ m and the distance between the first person A and the temple is $30\sqrt{3}$ m. [CBSE Question Bank]

Based on the above information, answer the following questions.

(i) $\angle CAB=\alpha=$

(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$

(c) $\sin^{-1}(2)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\angle CAB=\alpha=$

(a) $\cos^{-1}\left(\frac{1}{5}\right)$ (b) $\cos^{-1}\left(\frac{2}{5}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{4}{5}\right)$

(iii) $\angle BCA=\beta=$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$

(c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\tan^{-1}(\sqrt{3})$

(iv) $\angle ABC=$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(v) Domain and Range of $\cos^{-1}x=$

(a) $(-1,1),(0,\pi)$ (b) $[-1,1],(0,\pi)$

(c) $[-1,1],[0,\pi]$ (d) $(-1,1),\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

| ANSWERS |

1. (a) 2. (d) 3. (c) 4. (b) 5. (a)

6. (b) 7. (a) 8. $\frac{5\pi}{6}$ 9. $\frac{-\pi}{4}$

10. (i) $\frac{\pi}{6}$, (ii) $\frac{\pi}{6}$, (iii) $-\frac{\pi}{4}$, (iv) $\frac{\pi}{3}$, (v) $\frac{\pi}{6}$, (vi) $\frac{\pi}{6}$, (vii) $\frac{2\pi}{3}$, (viii) $\frac{5\pi}{6}$

11. $x\in[0,1]$ 12. $\frac{\pi}{6}$ 13. π 14. $\frac{5\pi}{6}$ 15. $\left(-\frac{3\pi}{2},-\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

16. $-\frac{\pi}{3}$ 17. $\frac{3\pi}{4}$ 18. $\frac{5\pi}{6}$ 19. $\sqrt{3}$

20. (i) → (b), (ii) → (c), (iii) → (d), (iv) → (c), (v) → (c)

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